Teaching Notes and In-Class Examples:

Section 7.2: Systems of Linear Equations in Two Variables:

These are the notes that I will teach from. Some examples will be worked as a class on the board, others I will give the student’s to work on independently. These examples are marked below and will count as the guided practice for this lesson. For the independent problems, I will walk around the room assisting the students and making sure everyone has a grasp on the concept before moving on. At the end of the lesson I will give a short worksheet and homework assignment. All examples and answers are provided below.

**The Method of Elimination:**

Objective: Use the method of elimination to solve systems of linear equations in two variables.

Steps:

1. Obtain coefficients for one of the variables that only differ in sign by multiplying all the terms of one or both equations by a suitable constant.
2. Add the equations to eliminate one variable; solve for remaining variable.
3. Substitute the value obtained back into one of the original equations and solve for other variable.
4. Check your solution by plugging the values back into both of the original equations.

Example: 3x + 5y = 7

 - 3x – 2y = -1

 0x + 3y = 6

 y = 2

 Now you can plug this y-value back into one of the original equations and solve for x.

 3x + 5(2) = 7

 3x = -3

 x = -1

**Examples to Work as a Class:**

1. Solve the system of linear equations by elimination.

3x + 2y = 4

5x – 2y = 8

Solution: Eliminate y-terms by adding equations:

 3x + 2y = 4

 + 5x – 2y = 8

 8x = 12

 x = 1.5

 Substitute the value of x back into the original equation to get y-value:

 3x + 2y = 4

 3(1.5) + 2y = 4

 2y = -0.5

 y = -0.25

The students can now check their answers by plugging the values for x and y back into the original two equations:

 3(1.5) + 2(-0.25) = 4?

 4.5 – 0.5 = 4 YES

 5(1.5) – 2(-0.25) = 8?

 7.5 + 0.5 = 8 YES

So the solution is (1.5, -0.25)

1. Solve the system of linear equations.

5x + 3y = 9

2x – 4y = 14

Solution:

We will have to multiply each equation by a constant to obtain a variable that only differs by a sign. Then we can add the equations.

4(5x + 3y = 9) → 20x + 12y = 36

3(2x – 4y = 14) → + 6x – 12y = 42

 26x = 78

 x = 3

Substitute this back into one of the original equations to get the y-value.

 5(3) + 3y = 9

 15 + 3y = 9

 3y = -6

 y = -2

Check your solution:

 5(3) + 3(-2) = 9

 15 – 6 = 9 YES

 2(3) – 4(-2) = 14

 6 + 8 = 14 YES

So the solution is (3, -2).

**Examples for the Students to Work in Class Independently:**

Give these problems and walk around the room assisting students and making sure they have a good grasp of the concept before moving on to the next objective.

1. 3x – 2y = 5 2. x + 7y = 12

 x + 2y = 7 3x – 5y = 10

Answers: 1. (3, 2)

 2. (5, 1)

**Graphical Interpretation of Two-Variable Systems:**

Objective: Graphically interpret the number of solutions of a system of linear equations in two variables.

There are three possible interpretations:

One solution → The two lines intersect at exactly one point.

Infinitely many solutions → The two lines are coincident (identical).

No solutions → The two lines are parallel.

A system is consistent if it has at least one solution, and inconsistent if it has no solution.

**Examples to Work as a Class:**

1. Determine the number of solutions for each system and whether the system is consistent or inconsistent. (In class graphs will be drawn on board.)

1. -2x + 3y = 6 This system of equations has no solution and 4x – 6y = -3 is inconsistent. The lines are parallel.
2. -2x + 3y = 6 This system of equations has infinitely many

-6x + 9y = 18 solutions, because the equations are the

 same line. Multiplying equation 1 by 3

 creates equation 2. The system is

 consistent.

1. -2x + 3y = 6 This system of equations has one solution

 x + 3y = 15 and is consistent.

2. Solve the system of linear equations. Determine the number of solutions and whether the system is consistent or inconsistent. Then verify by graphing the system of equations.

 x – 2y = 3

 -2x + 4y = 1

 Solution: Multiply to get coefficients that only differ by a sign. Then add the equations.

 2(x – 2y = 3) → 2x – 4y = 6

 -2x + 4y = 1 → + -2x + 4y = 1

 0 + 0 = 7 NO!!!

 Since this is a false statement, the system in inconsistent and has no solutions.

3. Solve the system of linear equations. Determine the number of solutions and whether the system is consistent or inconsistent. Then verify by graphing the system of equations.

 2x – y = 1

 4x – 2y = 2

 Solution: Multiply to get coefficients that only differ by a sign. Then add the equations.

 -2(2x – y = 1) → -4x + 2y = -2

 4x – 2y = 2 → + 4x – 2y = 2

 0 + 0 = 0 so 0 = 0

This is true for all values of x and y, so that the equations are equivalent. This means that the lines are identical and the system has infinitely many solutions. The system is consistent.

**Additional Trick and Example:**

 Trick: Sometimes these linear equations have decimal coefficients. When this is the case, you can multiply the equations by 10, 100 or 1000 to produce whole number coefficients.

 .02 x 100 = 2

 .143 x 1000 = 143

 Example: Solve the system of linear equations.

 0.02x – 0.05y = -0.38

 0.03x + 0.04y = 1.04

Solution: The coefficients have two decimal places, so you can multiply each equation by 100 to get whole number coefficients. Then add the equations.

 100(0.02x – 0.05y = -0.38) → 2x – 5y = -38

 100(0.03x + 0.04y = 1.04) → 3x + 4y = 104

Now multiply the equations by constants to obtain coefficients that only differ by a sign. Then add the equations and solve for the variable.

 3(2x – 5y = -38) → 6x – 15y = -114

 -2(3x + 4y = 104) → + -6x – 8y = -208

 0 – 23y = -322

 y = 14

Now plug the y-value into one of the original equations and solve for x.

 3x + 4(14) = 104

 3x + 56 = 104

 3x = 48

 x = 16

Have the students check their answers by plugging both values into both of the original equations. This will verify that the solution to this system of equations is (16, 14).

Extension: Ask the students to describe the solution and whether or not the system is consistent or inconsistent. Then have them graph the system to verify their answers.

**Examples for the Student’s to Work in Class Independently:**

1. 2/3x + 1/6y = 2/3

 4x + y = 4

1. 6x – 5y = 3

-12x + 10y = 5

1. 4x + 3y = 3

3x + 11y = 13

1. 6.3x + 7.2y = 5.4

5.6x + 6.4y = 4.8

1. 0.2x + 0.4y = -0.2

 x + 0.5y = -2.5

Answers: 1. Infinitely many solutions; consistent; same line

 2. No solution; inconsistent; parallel lines

 3. One solution: (-6/35, 43/35); consistent; intersecting lines

 4. Infinitely many solutions; consistent; same line

 5. One solution: (-3, 1); consistent; intersecting lines

**Application:**

Objective: Use systems of linear equations in two variables to model and solve real-life problems.

Ask yourself the question “How can I tell when to use systems of linear equations to solve application problems?”

Conditions:

1. Read the problem and determine whether or not the problem involves more than one unknown quantity.
2. Determine whether or not there are two (or more) equations or conditions to be satisfied.

If these conditions are met, then try setting up the problem as a system of linear equations.

**Example to be Done as a Class:**

An airplane flying into a headwind travels the 1650-mile flying distance between two cities in 3 hours and 18 minutes. On the return flight, the distance is traveled in 3 hours. Find the airspeed of the plane and the speed of the wind, assuming that both remain constant.

Solution: Find the unknown quantities:

 1. The speed of the wind = r1

 2. The speed of the plane = r2

 Obtain equations for these unknowns:

 r1 – r2 = the speed of the plane against the wind

 r1 + r2 = the speed of the plane with the wind

 Use the formula distance = (rate)(time) to obtain new equations:

 1650 = (r1 – r2)(3 + 18/60)

 1650 = (r1 + r2)(3)

 Simplify and add the equations:

 1650 = (r1 – r2)(3.3) → 500 = r1 – r2

 1650 = (r1 + r2)(3) → + 550 = r1 + r2

 1050 = 2r1

 525 = r1

 Substitute the value of r1 back into one of the original equations to get the value of r2.

 1650 = (525 + r2)(3)

 550 = 525 + r2

 25 = r2

 Check your work to verify these answers.

 Answers: The airspeed = 525 miles per hour

 The wind speed = 25 miles per hour

**Example for the Students to Work Independently:**

Revenues for a video rental store on a particular Friday evening are $867.50 for 310 rentals. The rental fee for movies is $3.00 each and the rental fee for video game cartridges is $2.50 each. Determine the number of each type that are rented that evening.

Answer: 185 movies and 125 video game cartridges.